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Development and validation of an algorithm for determining the parameters of pile foundations of bridge supports based on the finite element method

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Abstract

The article describes a complex algorithm developed by the authors for the automated determination of key design parameters of pile foundations of bridge supports, such as the level of conditional sealing and lateral ground pressure diagrams. The algorithm is based on the finite element method (FEM) and is intended for software implementation as part of one of the modules of the PARIS software package, designed to perform calculations of building structures, including bridge structures. The relevance of the work is due to the need to reduce the complexity of calculations of pile foundations and the risks of errors due to the human factor by automating routine processes in comparison with the analytical methods given in the "Code of Rules 24.13330.2021 Pile foundations" without reducing the reliability of the results obtained. The paper describes in detail the mathematical formulation of the problem, the structure of the algorithm, and the methodology for interpreting discrete reactions of the FEM model into standard values. The developed approach was validated on a series of test tasks, including single piles and bush foundations in various engineering and geological conditions. The results confirm the high convergence of the proposed method with analytical solutions while reducing labor intensity. Thus, the developed algorithm is a step towards more efficient design of pile foundations, combining the speed of numerical calculation methods with the requirements of current design standards.

Keywords: bridge crossing, pile foundation, finite element method, PARIS calculation complex, design automation, calculation automation, numerical modeling

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Техникалық ғылымдар. Құрылыс

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Ақырлы элементтер әдісіне негізделген көпір тіректерінің қадалық іргетастарының параметрлерін анықтау алгоритмін әзірлеу және тексеру

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Түйіндеме

Мақалада авторлар жасаған көпір тіректерінің қадаларының негізгі есептік параметрлерін автоматтандырылған анықтаудың кешенді алгоритмі сипатталған, мысалы, шартты тығыздау деңгейі және топырақтың бүйірлік қысымының диаграммалары. Алгоритм ақырлы элементтер әдісіне (МКЭ) негізделген және құрылыс конструкцияларының, оның ішінде көпір құрылыстарының есептеулерін орындауға арналған Париж бағдарламалық кешені модульдерінің бірінің құрамында бағдарламалық іске асыруға арналған. Жұмыстың өзектілігі "24.13330.2021 ережелер жинағында" келтірілген аналитикалық әдістермен салыстырғанда, алынған нәтижелердің дұрыстығын төмендетпестен, әдеттегі процестерді автоматтандыру арқылы қадалардың іргетастарын есептеудің күрделілігін және адам факторына байланысты қателіктер қаупін азайту қажеттілігімен байланысты. Жұмыста есептің математикалық тұжырымы, алгоритмнің құрылымы және мкэ-модельдің дискретті реакцияларын нормативтік шамаларға интерпретациялау әдістемесі егжей-тегжейлі сипатталған. Әр түрлі инженерлік-геологиялық жағдайларда бір қадалар мен бұталы іргетастарды қамтитын бірқатар тест тапсырмаларында әзірленген тәсілді валидациялау жүргізілді. Нәтижелер ұсынылған әдістің аналитикалық шешімдермен жоғары конвергенциясын растайды, сонымен бірге еңбек сыйымдылығын төмендетеді. Осылайша, әзірленген алгоритм сандық есептеу әдістерінің жылдамдығын қолданыстағы жобалау нормаларының талаптарымен біріктіретін қадалардың іргетастарын тиімдірек жобалауға қадам болып табылады.

Түйін сөздер: есептеулерді алгоритмдеу, қадалар негіздері, ақырлы элементтер әдісі, ПАРИС бағдарламалық кешені, дизайнды автоматтандыру, сандық модельдеу

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Технические науки. Строительство

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Разработка и валидация алгоритма определения параметров свайных фундаментов мостовых опор на основе метода конечных элементов

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Аннотация
В статье описан разработанный авторами комплексный алгоритм автоматизированного определения ключевых расчетных параметров свайных фундаментов мостовых опор, таких как уровень условной заделки и эпюры бокового давления грунта. Алгоритм базируется на методе конечных элементов (МКЭ) и предназначен для программной реализации в составе одного из модулей программного комплекса «ПАРИС», предназначенного для выполнения расчетов строительных конструкций, в том числе мостовых сооружений. Актуальность работы обусловлена необходимостью снижения трудоемкости расчетов свайных фундаментов и рисков ошибок из-за человеческого фактора с помощью автоматизации рутинных процессов по сравнению с приведенными в «Свод правил 24.13330.2021 Свайные фундаменты» аналитическими методиками без снижения достоверности получаемых результатов. В работе подробно описаны математическая постановка задачи, структура алгоритма и методика интерпретации дискретных реакций МКЭ-модели в нормативные величины. Проведена валидация разработанного подхода на серии тестовых задач, включающих одиночные сваи и кустовые фундаменты в различных инженерно-геологических условиях. Результаты подтверждают высокую сходимость предложенного метода с аналитическими решениями при одновременном снижении трудоемкости. Таким образом, разработанный алгоритм – шаг к более эффективному проектированию свайных фундаментов, сочетающий скорость численных методов расчета с требованиями действующих норм проектирования.

Ключевые слова: алгоритмизация расчетов, свайные фундаменты, метод конечных элементов, программный комплекс ПАРИС, автоматизация проектирования, численное моделирование

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1. Introduction

Pile foundations of bridge supports are among the most critical structural elements of transportation infrastructure, ensuring the transfer of permanent and temporary loads from superstructures to the underlying soil [1]. The reliability, serviceability, and durability of bridge structures largely depend on the accurate assessment of the stress–strain state of the “structure–foundation–soil” system. Therefore, improving the methods used for the analysis and design of pile foundations remains an important task in bridge engineering [2].

Current engineering practice relies on the requirements of design standards, such as SP 24.13330.2021 “Pile Foundations” [3], which are based on the theory of an elastic foundation (Winkler model). In this approach, the soil mass is represented by a system of independent elastic springs, allowing the interaction between the pile and the surrounding ground to be described analytically. Despite the widespread application and regulatory validity of these methods, their practical implementation is associated with considerable computational complexity. The determination of design parameters requires iterative procedures, extensive use of tabulated influence coefficients, consideration of multilayer soil profiles, and assessment of pile group interaction effects [4,5].

Particularly labor-intensive is the determination of design parameters required for subsequent checks of bearing capacity and stability, including the depth of conditional embedment and the distribution of lateral soil pressure. Manual calculations involve numerous intermediate operations and repeated iterations, which increase both the time required for analysis and the probability of errors caused by the human factor, especially when several design alternatives and geological conditions are considered [6].

In recent years, the increasing development of numerical methods and computational technologies has created new opportunities for the automation of geotechnical calculations. The finite element method (FEM) enables the simulation of complex spatial interactions between piles and the surrounding soil, taking into account the stiffness characteristics of structural elements, soil stratification, and load redistribution within pile groups. However, existing software implementations are primarily focused on obtaining displacements and internal forces and do not always provide engineers with the specific design parameters required by current standards for subsequent verification procedures [7,8].

Consequently, there is a need for specialized algorithms capable of automatically extracting standard design parameters from finite element models and ensuring consistency with conventional analytical approaches. The integration of such algorithms into engineering software can significantly reduce computational effort, improve the reliability of design solutions, and minimize the influence of human errors.

The aim of this study is to develop and validate an algorithm for the automated determination of design parameters of bridge pile foundations based on the finite element method for subsequent implementation within the PARIS software package.

To achieve this objective, the following tasks are addressed:

1. To establish the mathematical relationship between discrete reactions obtained from the finite element model and the design parameters used in the analytical methodology;
2. To develop an algorithm for determining the depth of conditional embedment and lateral soil pressure based on numerical calculations;
3. To implement the proposed approach in the PARIS software environment;
4. To verify the accuracy of the developed algorithm by comparing the results of numerical simulations with analytical calculations performed according to SP 24.13330.2021.

The scientific and practical significance of this research lies in the development of an algorithmic approach that combines the efficiency of finite element analysis with the requirements of current design standards, thereby increasing the productivity of engineers and improving the reliability of pile foundation design for bridge structures.

2. Materials and methods

2.1 Theoretical foundations of analytical calculation

Unlike vertical loads, where axial deformations predominate, the behaviour of piles under the action of horizontal forces and moments is described by a complex spatial model of the interaction between the structure and the soil mass. The calculation requires simultaneous consideration of the pile geometry, their layout in plan, the physical and mechanical properties of the soils, and the characteristics of the pile and grillage materials; furthermore, where there are several piles in a row, a group interaction effect arises that alters the load distribution [9,10].

The material presented below systematises the complete calculation cycle for pile foundations under horizontal loads in accordance with the requirements of SP 24.13330.2021[3] and includes the determination of the design pile width (b_p), the depth of the pile's effective embedment (l_1) and the pile deformation coefficient, taking into account the multi-layered nature of the soil, as well as the calculation of lateral soil pressure.

The soil pressure on the base of the pile (casing or column) is taken into account based on the actual dimensions of the base, whilst the pressure on the lateral surface is treated as that acting on a free-standing pile of rectangular cross-section operating under plane-stress conditions, with a design width of b_p. The difference between the actual operating conditions of the piles and those assumed in deriving the formulae is reflected by special experimentally determined coefficients when converting from the actual thickness (diameter) of the pile$d$ to the design width b_p, calculated for a column or casing using the formula:

$$b_p = kk_{\text{ф}}k_0 d, \tag{1}$$

and for a pile – using the formula:

$$b_p = k_{\text{ф}}k_0 d, \tag{2}$$

where k is a coefficient accounting for the mutual influence of shells or piles arranged in a single row parallel to the plane of action of horizontal and moment loads on their behavior in the ground; $k_{\text{ф}}$ is a coefficient accounting for the shape of the pile's cross-section (for a square cross-section $k_{\text{ф}} = 1{,}0$, for a circular cross-section $k_{\text{ф}} = 0{,}9$); k_0 — a coefficient accounting for the difference between the actual (spatial) operating conditions of the pile and the conditions of a plane problem**.**

For shells or columns, the coefficient k_0 is determined by the formula:

$$k_0 = 1 + \frac{1}{d}, \tag{3}$$

and for piles – using the formula:

$$k_0 = 1{,}5 + \frac{0{,}5}{d}, \tag{4}$$

where d is the thickness (diameter) of the pile, shell or column, m.

Thus, the formulae for calculating the design width b_p (in m) take the form:

a) for shells or columns

$$b_p = kk_{\text{ф}}(d + 1), \tag{5}$$

b) for piles

$$b_p = k_{\text{ф}}(1{,}5d + 0{,}5), \tag{6}$$

The coefficient k is taken to be equal to

$$k = k_1 + \frac{(1 - k_1)L_p}{2(d + 1)}. \tag{7}$$

where k_1 – a coefficient depending on the number n_p of shells or columns in a single vertical plane (in a single row) parallel to the plane of the applied load (when n_p=1, k_1 =1,0; when n_p=2, k_1=0,6); when n_p=3, k_1 =0,5; when n_p ≥4, k_1=0,45); L_p — the average clear distance (at ground level) between the shells or columns located in the plane under consideration, m.

The depth of conditional embedment is not merely a geometric parameter, but a design depth that separates two different modes of structural behavior: below this level, the pile is considered to be fully embedded in the ground, whilst above it, it behaves as a beam on an elastic foundation, resisting bending moments.

$$l_1 = l_0 + \frac{2}{\alpha_c}, \tag{8}$$

where l_0 – the distance from the top of the pile to the design ground surface; α_c – the deformation coefficient, calculated using the formula:

$$\alpha_c = \sqrt[5]{\frac{Kb_p}{EJ_c}}, \tag{9}$$

where E is the initial modulus of elasticity of the pile concrete under compression and tension, MPa, taken in accordance with SP 35.13330.2011; I_c is the moment of inertia of the pile cross-section, m^4; K is the soil proportionality coefficient.

If there are several soil layers with different characteristics within the pile embedment depth, the reduced values of the proportionality coefficient K are introduced into the calculation. In this case, soil layers located within a height h_k from the design ground level are taken into account:

$$h_k = 3{,}5d + 1{,}5. \tag{10}$$

If, with in a depth h_k, measured from the design ground level (d — the diameter of a circular pile or the side of a pile with a rectangular cross-section in the plane of the applied loads), there are two soil layers, then the reduced value *of K* is calculated using the formula:

$$K = \frac{K_1 h_{c1}(2h_k - k_{c1}) + K_2(h_k - h_{c1})^2}{h_k^2}, \tag{11}$$

and if there are three layers, using the formula:

$$K = \frac{K_1 h_{c1}[2(h_{c3} + h_{c2})] + K_2 h_{c2}(2h_{c3} + h_{c2}) + K_3 h_{c3}^3}{h_k^2}. \tag{12}$$

where h_{c1}, h_{c2}, h_{c3} are, respectively, the thicknesses of the first, second and third soil layers with in the depth h_k; K_1, K_2, K_3 are, respectively, the proportionality coefficients for the first, second and third soil layers. The values of the proportionality coefficients are taken from.

The most labour-intensive and complex process is the determination of lateral pressure. The design pressure on the soil σ_{zi}, kPa, at the lateral surface of the pile, determined at the reduced depth z = 0.85/α_c, is found using the formula:

$$\sigma_{zi} = Kz\left(A_1 u_{0i} - B_1 \frac{w_{0i}}{\alpha_\varepsilon} + C_1 \frac{M_{0i}}{\alpha_\varepsilon^2 EI} + D_1 \frac{H_i}{\alpha_\varepsilon^3 EI}\right), \tag{13}$$

The key difficulty in determining the design soil pressure lies in the need to perform complex iterative calculations. These are highly labour-intensive, as the calculations must sequentially account for the changing geometry of the foundation, the non-linearity of soil properties and foundation configurations, as well as a multitude of load combinations. Taken together, the calculation represents a complex, multi-factor problem [11].

The term outside the brackets KZ in Equation (13) represents the basis for determining the increase in soil resistance with depth in the linear model; inside the brackets is a linear combination of four initial parameters acting at the top of the pile: u_0(horizontal displacement), w_0(rotation angle), M_0 (bending moment) and H_0 (horizontal force). A_1, B_1, C_1, D_1 are dimensionless influence coefficients, which are functions of the dimensionless depth z. Their values are tabulated; the tables themselves are given in Appendix A of SP 24.13330.2021. These coefficients determine exactly how each of the initial parameters (displacement, rotation angle, moment, force) influences the magnitude of the pressure at a given depth.

To determine the lateral earth pressure, a sequence of calculation steps must be performed.

1. The displacements of the pile cross-section at the level of the design ground surface for a high grillage or at the level of the grillage base for a low grillage are determined from the unit forces applied to the pile at this level:

$$\varepsilon_{HH} = \frac{A_0}{\alpha_\varepsilon^3 EI};\ \varepsilon_{\mathrm{M}H} = \varepsilon_{\mathrm{M}H} = \frac{B_0}{\alpha_\varepsilon^2 EI};\ \varepsilon_{\mathrm{MM}} = \frac{\mathrm{C}_0}{\alpha_\varepsilon EI}, \tag{14}$$

where ε_{nn}, ε_{mn} – horizontal displacement and angle of rotation of the pile cross-section due to the action of a horizontal force H = 1 (Figure 1a); ε_{nm}, ε_{mm} – horizontal displacement and angle of rotation of the pile cross-section due to the action of a moment M = 1 (Figure 1b); A_0 , B_0 , C_0 – dimensionless coefficients taken from the tables in Appendix A of SP 24.13330.2021 depending on the reduced pile embedment depth in the ground $\bar{l}$, determined by the formula:

$$\bar{l} = \alpha_\varepsilon l, \tag{15}$$

where l is the length of the part of the pile embedded in the ground.

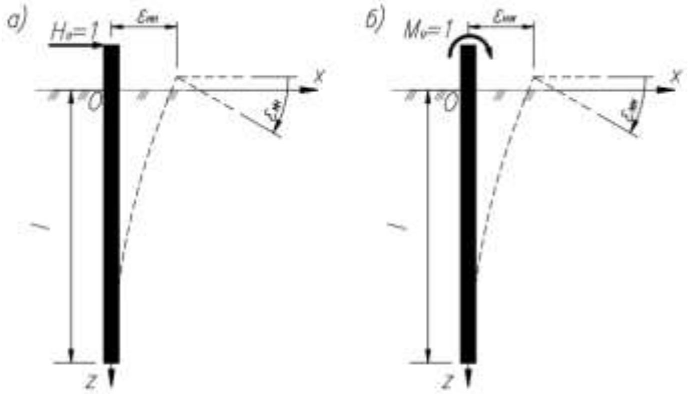

Figure 1. Diagrams of displacement caused by a unit force (a) and a unit moment (b) applied at ground level [author's material]

2. Calculate the displacements of the pile cross-section at the base of the grillage slab δ_{nn}, δ_{mn}– the horizontal displacement and angle of rotation of the pile section with a free upper end at the level of the grillage base due to a horizontal force H = 1 applied at the same level (Figure 2a), and δ_{nn}, δ_{mm} – the same for a moment M = 1 (Figure 2b) using the following formulae:

$$\delta_{HH} = \frac{l_0^3}{2EI} + \varepsilon_{\mathrm{MM}} l_0^2 + 2\varepsilon_{\mathrm{M}H} l_0 + \varepsilon_{HH};\ \delta_{MM} = \frac{l_0}{EI} + \varepsilon_{\mathrm{MM}},$$
$$\delta_{HM} = \frac{l_0^2}{2EI} + \varepsilon_{\mathrm{MM}} l_0 + \varepsilon_{\mathrm{M}H}. \tag{16}$$

In the case of designing a foundation with a low grillage $l_0 = 0$, therefore $\delta_{nn} = \varepsilon_{nn}$, $\delta_{mm} = \varepsilon_{mm}$, $\delta_{mn} = \varepsilon_{mn}$.

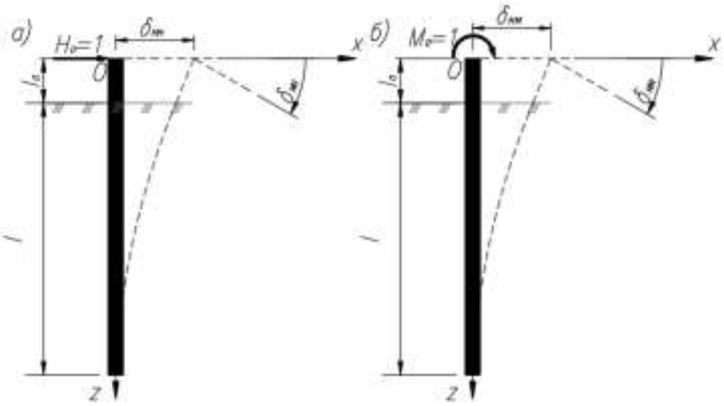

Figure 2. Displacement diagrams for a unit force (a) and a unit moment (b) applied at the level of the grillage slab base [author's material]

3. Determination of pile stiffness characteristics:

$$\rho_1 = \frac{EA}{l_N}; \rho_2 = \frac{\delta_{\text{мм}}}{\delta_{\text{нн}}\delta_{\text{мм}} - \delta_{\text{нм}}^2};$$
$$\rho_3 = \frac{\delta_{\text{нм}}}{\delta_{\text{нн}}\delta_{\text{мм}} - \delta_{\text{нм}}^2}; \rho_4 = \frac{\delta_{\text{нн}}}{\delta_{\text{нн}}\delta_{\text{мм}} - \delta_{\text{нм}}^2}. \quad (17)$$

where ρ_1 – the force acting along the pile axis on the grillage when the grillage is displaced by a unit in this direction (Figure 3a); ρ_2 – the force acting on the grillage in a direction perpendicular to the pile axis when the grillage is displaced by a unit in that direction (Figure 3b); ρ_3 – the moment acting from the pile on the grillage when it is displaced by one unit in a direction perpendicular to the pile axis (Figure 3b), and, based on the principle of reciprocity of reactions, the force acting on the grillage in a direction perpendicular to the pile axis when the grillage is rotated by one unit (Figure 3c); ρ_4 – the moment acting on the grillage when the grillage rotates by one unit (Figure 2.5c).

The actual length of the suspended piles in the calculations is replaced by the design length in compression $l_N = l_0 + \frac{7EA}{10^3 F_d}$, determined for small-diameter suspended piles.

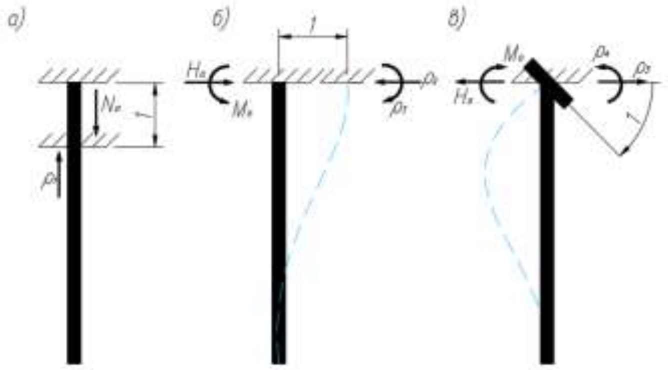

Figure 3. Unit displacements of the grillage and corresponding forces: a – during longitudinal displacement of the grillage; b – during transverse displacement of the grillage; c – at the angle of rotation of the grillage [author's material]

4. The coefficients of the canonical equations in the computational plane are calculated. The canonical equations of the displacement method for a symmetric planar circuit take the form:

$$r_{vv}v - F_v = 0;\ r_{uu}u + r_{u\omega}\omega - F_h = 0;\ r_{\omega u}u + r_{\omega\omega}\omega - M = 0,$$
$$r_{vv} = \rho_0 \sum \cos(\alpha_i)^2 + n\rho_2; \quad (18)$$

$$r_{uu} = \rho_0 \sum \sin(\alpha_i)^2 + n\rho_2;$$
$$r_{u\omega} = r_{\omega u} = \rho_0 \sum x_i \sin(\alpha_i)\cos(\alpha_i) - \rho_3 \sum \cos(\alpha_i);$$
$$r_{\omega\omega} = \rho_0 \sum {x_i}^2 \cos(\alpha_i)^2 + \rho_2 \sum {x_i}^2 + 2\rho_3 \sum x_i \sin(\alpha_i) + n\rho_4; \quad (19)$$

where $\rho_0 = \rho_1 - \rho_2$.

5. The vertical and horizontal displacements of the grillage, as well as its angle of rotation in the calculation plane, are calculated**:**

$$v = \frac{F_v}{r_{vv}};\ u = \frac{r_{\omega\omega}F_h - r_{u\omega}M}{r_{uu}r_{\omega\omega} - {r_{u\omega}}^2};\ \omega = \frac{r_{uu}M - r_{u\omega}F_h}{r_{uu}r_{\omega} - {r_{u\omega}}^2}. \quad (20)$$

where F_v, F_h, M – forces applied to the top of the pile.

6. The longitudinal force N_i, the transverse force H_i and the moment M_i acting at the point of connection with the grillage on each pile are determined (positive directions are shown in Figure 4):

$$N_i = \rho_1[u \sin\alpha_i + (v + x_i\omega)\cos\alpha_i];$$
$$H_i = \rho_1[u \cos\alpha_i + (v + x_i\omega)\sin\alpha_i] - \rho_3\omega; \quad (21)$$
$$M_i = \rho_4\omega - \rho_3[u \cos\alpha_i - (v + x_i\omega)\sin\alpha_i].$$

For vertical piles, the expressions in (21) are simplified and take the form:

$$N_i = \rho_1 u;\ H_i = \rho_2(v + x_i\omega) - \rho_2\omega;\ M = \rho_4\omega - \rho_3(v + x_i\omega). \quad (22)$$

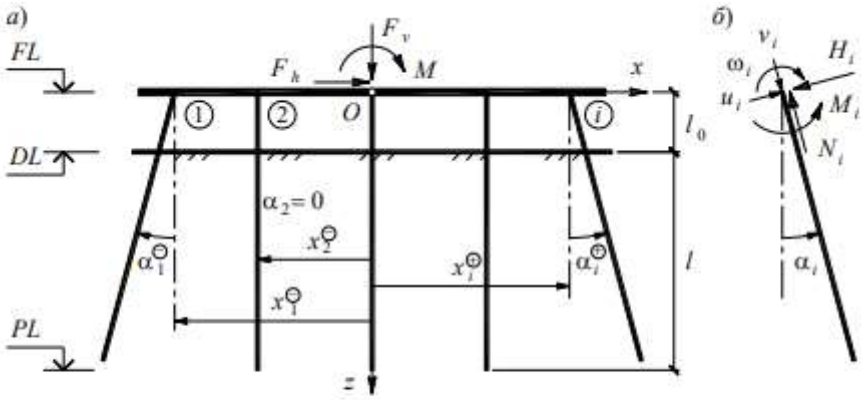

Figure 4. General diagram for the calculation of a pile foundation in the x0z coordinate system (a) and the positive directions of displacements and forces at the level where the pile is embedded in the grillage (b) [author's material]

7. To determine the required value, the initial forces and displacements must be calculated:

$$M_{0i} = M_i + H_i l_0;$$
$$u_{0i} = H_i\varepsilon_{\text{НН}} + M_{0i}\varepsilon_{\text{НМ}}; \quad (23)$$
$$\omega_{0i} = H_i\varepsilon_{\text{МН}} + M_{0i}\varepsilon_{\text{ММ}}.$$

The calculation of lateral pressure is the final stage, following the completion of all necessary computational procedures.

The method described, despite its accuracy and regulatory validity, is characterized by considerable computational complexity: the calculation requires the sequential application of more than twenty interrelated formulas, constant reference to tabulated coefficients from the SP appendices, and repeated execution of the entire calculation cycle when selecting the optimal foundation parameters. This multi-stage nature and the need to perform iterative procedures manually not only substantially increase the labour intensity of the design process but also heighten the risk of errors due to human error, making the automation of this calculation process a pressing issue.

2.2 Numerical approach

In numerical modelling, each pile within the foundation is represented in the calculation scheme as a beam-type finite element possessing bending, shear and axial stiffness. This approach allows for the bending of piles under horizontal loads (braking, wind, ice pressure), the interaction of piles within a group via the grillage, as well as the influence of the pile's length, cross-section and material on its deformability [12,13].

The soil surrounding the pile and located beneath its lower end is modelled using single-node elastic connections (also known as 'springs'), attached to the nodes of the pile's beam model [7, 9]. These connections simulate soil resistance both along the pile's lateral surface and beneath its tip.

In accordance with the recommendations of SP 24.13330.2011 'Pile Foundations' and the methodology proposed by N.M. Gersevanov and developed by V.A. Florin [14], the stiffness of such connections is determined on the basis of the bedding coefficient, which characterizes the specific reaction of the soil to a unit settlement.

To model the lateral resistance of the soil, the pile is divided into design sections of length l_i (usually 0.5–2.0 m, depending on soil heterogeneity). At each node corresponding to the boundary of a section, a horizontal elastic connection with stiffness k_i is established:

$$ki = Cxy \times dp \times hzi \tag{24}$$

The proposed approach has been implemented in the PARIS software package and is used in the inspection and design of existing bridges to assess their load-bearing capacity. The results obtained using this model show good agreement with field test data for pile foundations, which confirms its suitability for engineering practice.

A pile (column), as a support element, is a rod partially embedded in the ground, which is an elastoplastic deformable medium. In practical calculations, various assumptions are made to simplify the complex interaction between the pile and the elastoplastic soil medium [15].

To determine the level of conditional embedment, it is necessary to analyse the values of horizontal reactions R in the connections modelling the interaction of the pile with the soil. The distance from ground level to zero reaction, and consequently to zero horizontal displacement of the pile, is the level of conditional embedment l_1.

Instead of analytical relationships for determining lateral pressure, it is proposed to use directly the maximum reaction in a single-node connection R_{max}, obtained as a result of FEM calculation. The value of the lateral pressure is determined by dividing the reaction by the pressure distribution area, defined by the notional pile width b_p and the average distance between adjacent nodes of the FE mesh, according to the formula:

$$\sigma_z = \frac{Rmax}{b_p \cdot (\frac{l_i + l_{i+1}}{2})}. \tag{25}$$

2.3 Pile modelling

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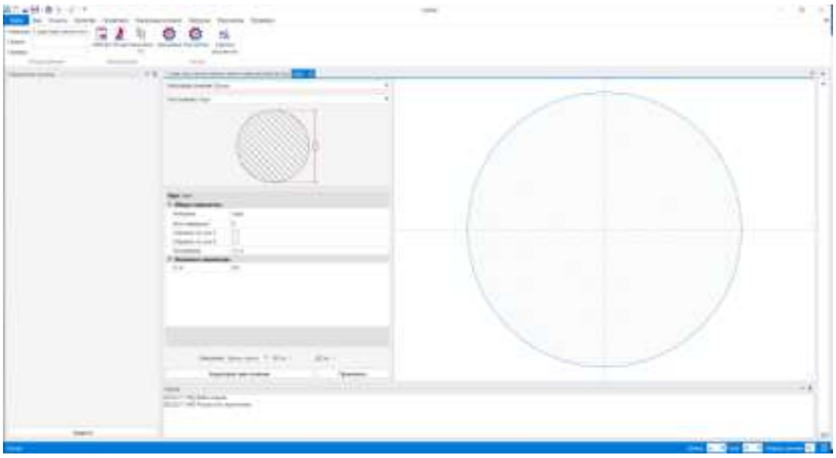
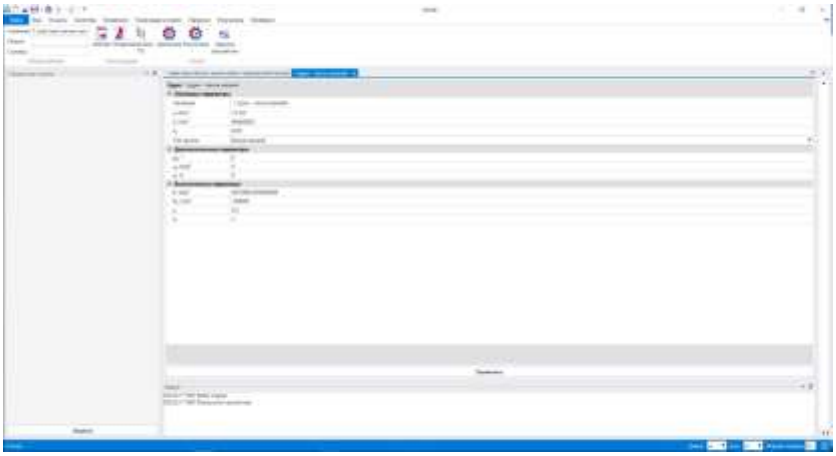
Modelling of pile foundations in the ‘PARIS’ software [16,18]. The first step involves creating the pile cross-section, its material, and the soil of each layer. To obtain accurate soil data, the main parameters of each soil layer must be entered; the programme then automatically calculates the proportionality coefficient *K*.



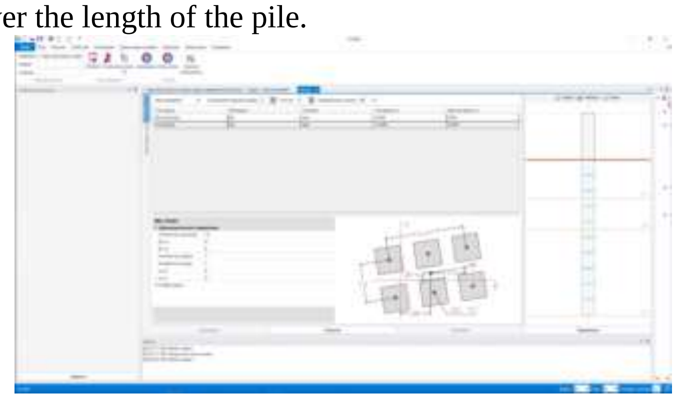
a



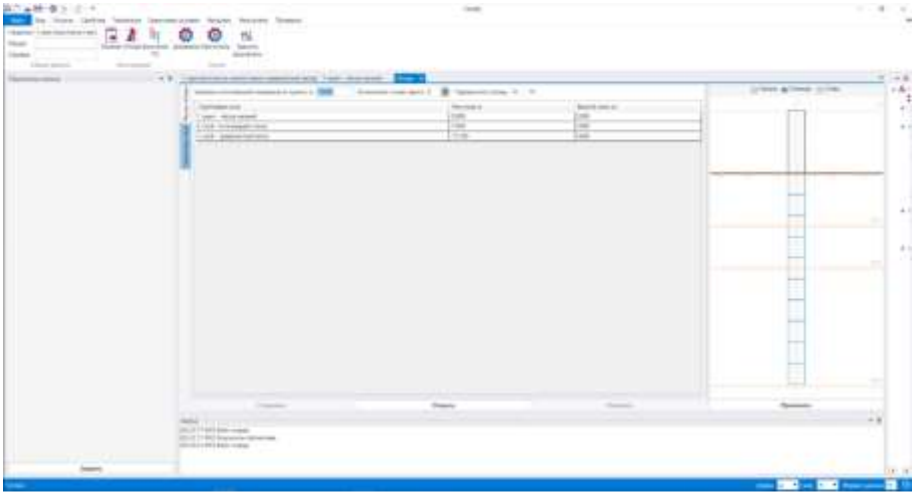
b

Figure 5. Development of the calculation model: a – creation of the pile cross-section; b – soil conditions [author’s material]

Next, the pile foundation itself is modeled in the ‘Support Wizard’ (fig.6), with two levels: the free length (from ground level to the top of the pile) and the length of the pile embedded in the soil. Each tier must be divided into finite elements; in this example, the length of the pile below ground was divided into 10 elements, and above ground into 5 elements. Next, in the ‘Soil Layers’ tab, the thickness of each layer must be specified, bearing in mind that the thickness of the bottom layer must completely cover the length of the pile.



A



b

Figure 6. Development of the pile foundation calculation model in the «Support Wizard»: (a) definition of pile geometry and finite element discretization; (b) assignment of soil layer parameters [author's material]

Once the input data has been entered correctly, a finished model of a single pile is generated. The contact between the pile element and the soil mass is modelled using a system of single-node connections. Once this interaction scheme has been created, the model is ready to be loaded with external forces.

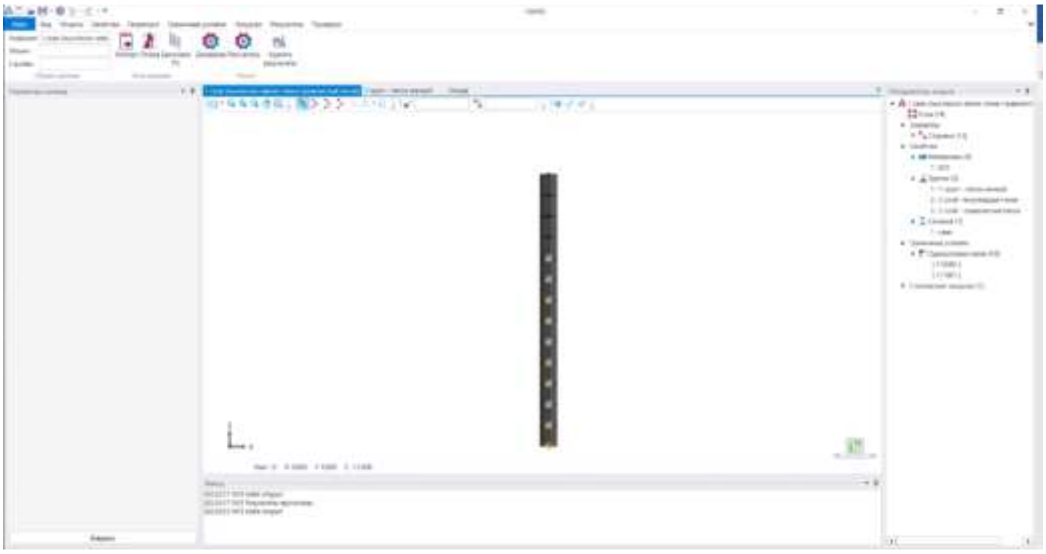


Figure 7. Final model of a single pile [author's material]

2.4 Validation of analytical and numerical calculations

To visually analyze the distribution of lateral pressure and determine the level of conditional embedment, a horizontal force H = 10 kN must be applied to the pile cap. Technically, this is achieved by creating the corresponding loading scenario in the calculation software. Once the force application has been correctly defined, the stress-strain state of the system is calculated.

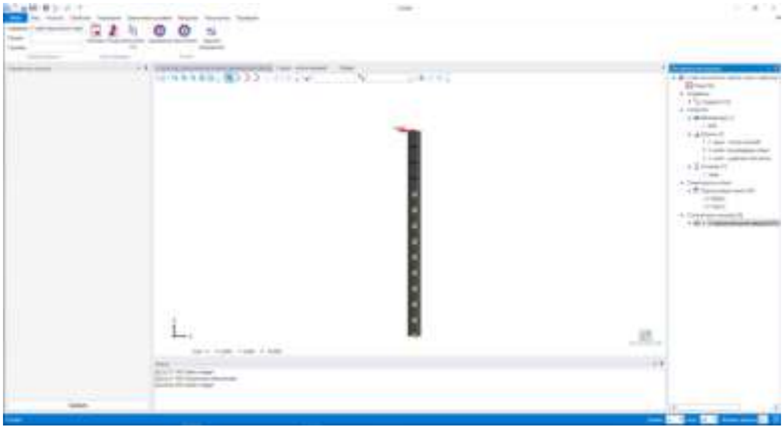


Figure 8. Application of a horizontal load [author's material]

Upon completion of the calculation, the values of the horizontal reactions in the single-node connections are obtained (Figure 9).

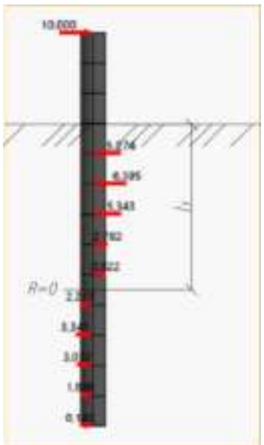


Figure 9. Distribution of horizontal reactions in single-node connections of a single pile [author's material]

To assess the collective interaction of piles in the ground, a cluster structure was modelled and analysed. It comprised two square-section piles (0.35×0.35 m, L=10 m) connected by a rigid grillage, which allowed the redistribution of loads within the system to be studied. For this system, the load was applied to the centre of gravity of the slab base.

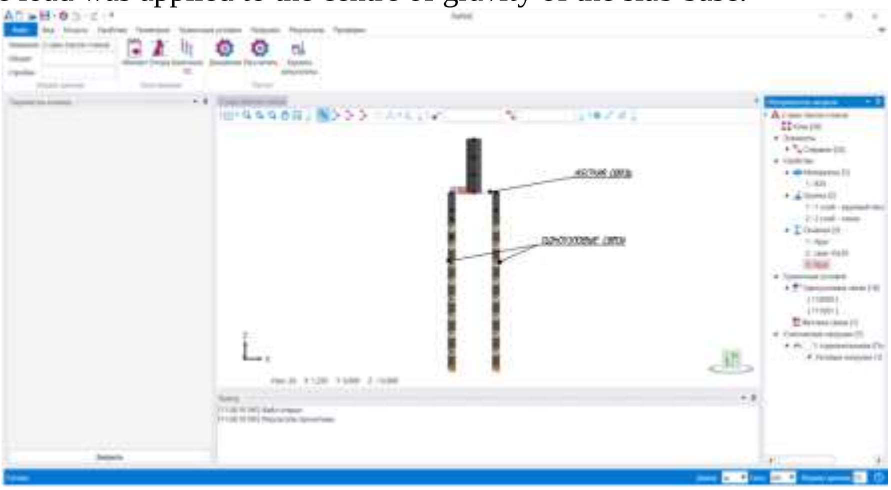


Figure 10. Model of a pile foundation consisting of a pile bundle [author's material]

3. Results

Two types of single pile models were considered: circular cross-section with a diameter of 0.6 m and square cross-section of 0.35×0.35 m. Soil conditions ranged from the simplest single-layer subgrade to a three-layer formation with varying mechanical properties, which allowed the influence of the complexity of the geological profile on the performance of pile structures to be assessed. The results of the analytical calculation according to SP 24.13330.2021 and the numerical simulation in the PARIS programme are presented in Table 1.

Table 1. Comparative analysis of analytical calculation and FEM for the values of the design embedment depth and lateral pressure

Soil	Values	Analytical calculation	Numerical calculation	Deviation
1 pile. Soil: coarse sand and clay (Ø0.8 m)	l_1	4.81 m	4.7 m	2.45%
	σ_z	3.85 kPa	4.11 kPa	6.25%
1 pile. Soil: Loam and plastic loam (Ø 0.8 m)	l_1	4.77 m	4.6 m	3.65%
	σ_z	4.36 kPa	4.79 kPa	9.02%
1 pile. Soil: silty sand, semi-cohesive clay, gravelly sand (Ø 0.8 m)	l_1	5.25 m	5.35 m	0.94%
	σ_z	3.72 kPa	3.95 kPa	5.80%
1 pile. Soil: Coarse sand (0.35 m x 0.35 m)	l_1	4.72 m	4.7 m	0.42%
	σ_z	6.26 kPa	6.52 kPa	3.98%

Two options for the arrangement of the pile group were considered: 2 piles along the bridge axis and 2 piles across the bridge axis, with identical geological data.

Table 2. Comparative analysis of analytical calculation and FEM for the design values of the conditional embedment depth and lateral pressure for the pile group

Soil	Values	Analytical calculation	Numerical calculation	Deviation
2 piles (0.35 m x 0.35 m). Soil: Coarse sand and clay (along the bridge axis)	l_1	3.81 m	4.1 m	7.07%
	σ_z	2.15 kPa	2.34 kPa	8.17%
2 piles (0.35 m x 0.35 m). Soil: Loam and plastic sandy loam (across the bridge axis)	l_1	2.90 m	3.20 m	6.56%

4. Discussion

A comparison of the analytical calculation based on SP 24.13330.2021 and the numerical simulation in the PARIS software package showed satisfactory agreement between the results. Deviations in the depth of the notional embedment (l_1) ranged from 0,4% to 7,1%, and in lateral pressure (σz) from 1,3% to 9,0%. In none of the cases considered did the deviations exceed 10%. Automating the calculation within the specialised software package reduces the time required to complete the full calculation cycle from 20 minutes (manual calculation with verification of tabulated coefficients) to a few minutes (model preparation, running the simulation, and analysing the results). Thus, the workload is reduced by a factor of 5–6, which is particularly significant when selecting foundation parameters from multiple variants.

A deviation of up to 10% between analytical and numerical methods is entirely acceptable for geotechnical calculations. This is due to the high variability of the physical and mechanical properties of soils (the error in determining soil properties often exceeds 20%), and the fundamental simplifications inherent in both methods (linear soil resistance model, idealisation of the ‘pile–soil’ interaction);

Both approaches can be applied in design practice, and their combined use allows for an increase in the reliability of the structural solutions adopted.

5. Conclusion

This study presents the development and validation of an algorithm for the automated determination of the key design parameters of bridge pile foundations based on the finite element method. The proposed approach has been implemented as a dedicated module within the PARIS software package and is intended to support the analysis and design of bridge foundations under various engineering and geological conditions.

The developed methodology establishes a mathematical relationship between the discrete reactions obtained from the FEM model and the continuous design parameters required by current design standards. This makes it possible to automatically determine the depth of conditional embedment and the distribution of lateral soil pressure without the need for labor-intensive analytical procedures and extensive use of tabulated coefficients.

The verification studies performed for single piles and pile groups demonstrated good agreement between the results obtained by numerical simulation and those calculated according to SP 24.13330.2021. The discrepancies between the analytical and numerical approaches did not exceed 9%, which is within the acceptable range for geotechnical engineering calculations and confirms the reliability and practical applicability of the proposed algorithm.

The implementation of the developed module significantly reduces the time required for pile foundation design, minimizes the probability of errors associated with manual calculations, and facilitates the optimization of structural solutions. In addition, the proposed approach increases the efficiency of engineering analysis and improves the reliability of design decisions for critical bridge structures.

Therefore, the developed algorithm represents an effective integration of finite element modelling with the requirements of current design standards. Its implementation in the PARIS software environment contributes to the digitalization and automation of bridge foundation design and provides a basis for the further development of intelligent computational tools for transport infrastructure engineering.

Future research may focus on extending the proposed methodology to nonlinear soil behaviour models, more complex pile group configurations, and the integration of optimization and machine learning techniques to enhance the automation and reliability of foundation design procedures.

Conflict of Interest. The corresponding author declares that there is no conflict of interest.

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